A Generalised Semantics for Belief Updates
—An Equivalence-based Approach

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Abstract. As suggested in the literature, revising and updating beliefs and knowledge bases is an important yet unsolved topic in knowledge representation and reasoning in Answer Set Programming (ASP) that requires a solid theoretical basis, particularly in current applications of Artificial Intelligence where an agent can work in an open dynamic environment with incomplete information. Various researchers have combined postulates and ASP as key components to set up their approaches. However, many of such proposals still present some shortcomings when dealing with persistence situations, redundant information, contradictions or they simply lack of further analysis of properties that should make them more accessible. In need to satisfy more general principles and a common frame of reference, this paper introduces a general framework for updates of logic programs, a properties characterisation and its equivalence with other variants. Rather than a sequence of updates of programs, this semantics consists in performing updates of epistemic states at the object level that meets well-accepted belief revision postulates and that follows the original AGM conception.

1 Introduction

One of the goals of Artificial Intelligence and in particular of commonsense reasoning is how to make an agent intelligent that may be autonomous and capable of acting in an open dynamic environment. As suggested in the logic-programming literature, such a goal requires a solid theoretical basis on knowledge representation and nonmonotonic reasoning, and in particular, in *knowledge updates*. Logic programming is a classical well-known mechanism to code and represent agents' knowledge by means of a set of clauses called logic program. Such a program might be called a knowledge base and we code it into a semantics called Answer Sets Programming [14] or ASP in short. However, logic programming has typically been static in the sense that it provides no mechanism to automatically make changes (belief revision or updates) to the knowledge base.

In particular, when updating knowledge one needs a way to avoid inconsistencies due to *potential contradictory information* upcoming from new evidence that is typically incomplete. Much work has been done in the context of logic

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programming based on a common ASP basis by satisfying certain properties and postulates: [1, 6, 12, 13, 21, 22, 23, 24]. However, despite the existence of several semantics for updates [3] and a vast analysis of general properties [13, 20], we claim we are still far from having a general one that can satisfy many existing and well-known principles to represent *"correct" dynamic knowledge.*

For instance, one of the missing and obvious properties in current semantics for updates is *persistence* that others don't manage well for several reasons, mainly for their approach on *sequences of updates* [3]. Such a problem has been introduced and overcome by Sakama and Inoue [22] by means of a semantics based on an extended particular version of abductive logic programming [15]. However, their semantics is strongly based on syntactical changes that has other problems (described later) and lacks of a proper characterisation of principles for updates (or belief revision), presumably due to their different goals of their referred approach.

With the aim to define a *general characterised semantics* and to succeed in the mentioned persistence situation (and in many others more), this paper consists of an alternate more-general approach, founded on generalised answer sets, and based upon further well-known principles for belief change (AGM-postulates [5]) that make it *syntax independent*, more intuitive and have generally-accepted properties.

Besides satisfying most belief revision postulates, this paper exhibits an approach that consists in performing *successive updates* so that the semantics can deal with the problem that, according to Sakama and Inoue, produces counter-intuitive interpretations in most approaches.

Partial results of this article have already appeared in preliminary versions (without proofs or other properties) in [2], as well as in the extended abstract in [4]

2   Problem Description

As introduced in previous paragraphs, once there existed a strong theoretical *basis for belief revision and updates*, a few authors in the field of logic programming proposed mechanisms for updates with a vast analysis, *taxonomy* and comparison of various known semantics [13, 22, 24]. Some others even suggested new principles, like [6, 7], with alternate logic-programming frameworks to ASP. However, owing to the different foundations each semantics has (even within ASP), yet some problems arise to meet a significant number of well-accepted *principles for updates* [3].

Although it is unlikely to come off with a semantics that may satisfy all of them [13, 20, 22], a more general one that fulfils most of widely-accepted principles is still necessary, which is not an easy task.

For instance, researchers studying updates of logic programs following principles for updates[1] (coded into eight well-known postulates for updates called

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[1] Actually the principles are for belief revision rather than updates. We explain the difference later.

AGM theory [5]) —and in other principles around it[2]— boil down to the difficulty in satisfying many of them by means of a *non-monotonic framework* like ASP, owing to the *monotonic nature* of the postulates themselves —[9, 13, 20, 22]. Nonetheless, Osorio and Cuevas [20] achieved an interpretation of six of the original eight AGM postulates in terms of the *monotonic* (non-classical) $\mathcal{N}_2$-logic, for general "update" operators. They have chosen (the monotonic) $\mathcal{N}_2$-logic apparently because it is one of several that characterise ASP and includes two types of negation: negation-as-failure and strong negation. The authors' results in terms of a general semantics, however, seem to be inconclusive [20].

On the other hand, the nearest proposal (to the best of our knowledge) that seemed[3] to meet most of the existing principles is due to Sakama and Inoue [22], who have introduced and overcome an interesting *persistence situation* that others fail to represent well for different reasons, as pointed out by themselves and in [2]. In particular, the main feature of that interesting situation is that Sakama and Inoue's semantics is capable of maintaining a *knowledge base* (coded into a logic program) throughout its own evolution, and that is also the main motivation behind A. Guadarrama [2] to propose his approach. They both lack, however, of a more-general belief-update characterisation, besides other problems.

Although their object-level approach makes it a good candidate to be considered by the belief-revision community, Sakama and Inoue's minimal-change principle is still *syntactic*: the *changes to a knowledge base* are to be minimal, as they themselves explain, and that has other problems. The main issue, however, in that they have no characterisation of their semantics with further general *belief-change principles*, arguing that they aren't applicable to nonmonotonic propositional theories in general [22].

In addition to that, a big disadvantage of *syntactical approaches*, as discussed in [13, 20] is that, in general, they do not satisfy the *structural properties* proposed in [20, 23], and Sakama and Inoue's approach is not an exception.

Regardless the polemic that approach might cause (especially in *planning* domains) and the deployment of extended-abduction properties in their article, the lack of further and more *general properties for belief change* makes it hard to compare with other alternatives for updates of logic programs. Indeed, their first goal (as they themselves explain) is the converse: to provide a mechanism of updates to characterise their *extended abduction framework* [22].

In need to define a *general semantics* and regardless the difference between belief revision and updates due to Katsuno and Mendelzon [18], this paper includes further results and a slightly different and fundamental approach from the preliminary alternate basic solution in [2, 4] within the same studied foundation of *Minimal Generalised Answer Sets* (MGAS hereafter, from Kakas and Mancarella [16]) and with an alternate approach to both Zacarías et al. [23] and A. Guadarrama [1], proposing a *simpler general formulation* that likewise per-

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[2]   For a nice analysis and compilation of such principles, see [13].

[3]   To my knowledge, there is no evidence that their semantics satisfies most of them, although it does overcome most of the problems that other semantics present.

forms multiple updates, but at the object level rather than sequences of updates, which overcomes the kind of problems already described. Moreover, the simpler semantics meets the *structural properties for updates* proposed by [13, 20, 23], as well as the *satisfaction of five of the six most general belief revision postulates* and many other relevant properties.

3 Preliminaries

A main foundation of this proposal is the well-known AGM-postulates [5] in a particular interpretation and notation [4], followed by a brief basic background of Answer Sets and Generalised Answer Sets. Owing to space constraints, however, this paper excludes sections of three ASP's characterising logic systems (intuitionistic logic, Nelson's logic and $\mathcal{N}_2$), which are more evidence of the solid foundation earlier suggested. Finally, in this paper it is assumed that the reader is familiar with basic notions of AGM-theory, as well as logic programming and in particular with ASP.

3.1   Logic Programming and Answer Sets

The following formalism gives the description of ASP, which is identified with other names like *Stable Logic Programming* or *Stable Model Semantics* [14] and A-Prolog. Its formal language and some more notation are introduced as follows.

Definition 1 (ASP Language of logic programs, $\mathcal{L}_{ASP}$). *In the following* $\mathcal{L}_{ASP}$ *is a language of* propositional logic *with* propositional symbols*:* $a_0, a_1, \ldots$ *;* connectives*: ",." (*conjunction*) and meta-connective ";"; disjunction, denoted as* "$|$"; $\leftarrow$ *(*derivation, *also denoted as* $\rightarrow$*);* propositional constants $\perp$ *(*falsum*);* $\top$ *(*verum*);* "$\neg$" *(*default negation *or* weak negation, *also denoted with the word* not*);* "$\sim$" *(*strong negation, *equally denoted as* "$-$"*);* auxiliary symbols*:* "(", ")" *(*parentheses*). The propositional symbols are called* atoms *too or* atomic propositions. *A* literal *is an atom or a* strong-negated atom. *A* rule *is an ordered pair* $Head(\rho) \leftarrow Body(\rho)$.

An intuitive meaning of *strong negation* "$\sim$" in logic programs with respect to the default negation "$\neg$" is the following: a rule $\rho_0 \leftarrow \neg\rho_1$ allows to derive ρ_0 when there is no *evidence* of ρ_1, while a rule like $\rho_0 \leftarrow \sim\rho_1$ derives ρ_0 only when there is an *evidence* for $\sim\rho_1$, i.e. when it can be proved that ρ_1 is false.

With the notation introduced in Definition 1, one may construct clauses of the following general form that are well known in the literature.

Definition 2 (EDLP). *An* extended disjunctive logic program *is a set of rules of form*

$$\ell_1 \vee \ell_2 \vee \ldots \vee \ell_l \leftarrow \ell_{l+1}, \ldots, \ell_m, \neg\ell_{m+1}, \ldots, \neg\ell_n \tag{1}$$

where ℓ_i *is a literal and* $0 \leq l \leq m \leq n$.

Naturally, an *extended logic program* (or ELP hereafter) is a finite set of rules of form (1) with $l = 1$; while an *integrity constraint* (also known in the literature as *strong constraint*) is a rule of form (1) with $l = 0$; while a *fact* is a rule of the same form with $l = m = n$. In particular, for a literal ℓ, the *complementary literal* is $\sim\ell$ and vice versa; for a set $\mathcal{M}$ of literals, $\sim\mathcal{M} = \{\sim\ell \mid \ell \in \mathcal{M}\}$, and $Lit_{\mathcal{M}}$ denotes the set $\mathcal{M} \cup \sim\mathcal{M}$; finally, a *signature* $\mathfrak{L}_\Pi$ is a finite set of literals occurring in Π. Additionally, given a set of literals $\mathcal{M} \subseteq \mathcal{A}$, the complement set $\overline{\mathcal{M}} = \mathcal{A} \setminus \mathcal{M}$.

Although we have introduced ASP as propositional (*ground*) programs, fixed *non-ground ASP-programs* of arbitrary *arity* are also considered in the same way than Dantsin et al. [10] do. Accordingly, non-ground ASP-programs with variables or constants as *arguments* are seen as a simplified expressions of larger ground (propositional) ones without variables, where each *ground program Π* is a set of its *ground rules* $\rho \in \Pi$. In addition, a ground rule is the set obtained by all possible substitutions of variables in ρ by constants occurring in Π [10].

Although ASP is a strong theoretical framework to represent knowledge, it can't model changes to that knowledge by itself, nor can it represent conflict situations amongst an agent and its dynamic environment. So, a more general and relaxed semantics is necessary, to choose potential models amongst *conflicting information*, which ought to reflect the general *principles* and *postulates* under consideration. In particular, a suitable framework to our purposes has been *Abductive Logic Programming* due to Kakas and Mancarella and is briefly presented in the following section.

3.2   Abductive Programs and MGAS

As one of the semantics to interpret abductive programs, *Minimal Generalised Answer Sets* (MGAS) provides a more general and flexible semantics than standard ASP, with a wide range of applications. This framework is briefly introduced in the following set of definitions.

Definition 3 (16). *An* abductive logic program *is a pair $\langle \Pi, \mathcal{A}^* \rangle$ where Π is an arbitrary program and $\mathcal{A}^*$ a set of literals, called abducibles.*

On the other hand, there already exists a semantics to interpret abductive programs, called *generalised answer sets* (GAS) due to Kakas and Mancarella.

Definition 4 (GAS, 16). *The expression $\mathcal{M}(\Delta)$ is a* generalised answer set *of the abductive program $\langle \Pi, \mathcal{A}^* \rangle$ if and only if $\Delta \subseteq \mathcal{A}^*$ and $\mathcal{M}(\Delta)$ is an answer set of $\Pi \cup \{\alpha \leftarrow \top \mid \alpha \in \Delta\}$.*

In case there are more than one generalised answer sets, a *preferred inclusion order* may be established:

Definition 5 (8). *Let $\mathcal{M}(\Delta_1)$ and $\mathcal{M}(\Delta_2)$ be generalised answer sets of $\langle \Pi, \mathcal{A}^* \rangle$. The relation $\mathcal{M}(\Delta_1) \leq_{\mathcal{A}^*} \mathcal{M}(\Delta_2)$ holds if and only if $\Delta_1 \subseteq \Delta_2$.*

Last, one can easily establish the *minimal generalised answer sets* from an abductive inclusion order with the following definition

Definition 6 (MGAS, 8). *Let $\mathcal{M}(\Delta)$ be a minimal generalised answer set (MGAS) of $\langle \Pi, \mathcal{A}^* \rangle$ if and only if $\mathcal{M}(\Delta)$ is a generalised answer set of $\langle \Pi, \mathcal{A}^* \rangle$ and it is minimal with respect to its abductive inclusion order.*

4   Updating Epistemic States

One of the main goals of this proposal is to meet most well-accepted *principles* for updates at the *object level* and in Minimal Generalised Answer Sets (MGAS), besides other relevant properties. The approach consists in setting up the needed models for the desired properties in an *iterated fashion*, rather than a sequence of updates, as earlier explained.

A first analysis of the problem at the object level, a solution, *justification, basic model-oriented properties* and *comparison with other semantics* are available in [2]. However, the semantics hasn't been characterised with more general principles, which is necessary both to avoid counterintuitive behavuour and to provide a common *frame of reference* to compare with other approaches. So, let us briefly introduce it, followed by a *characterisation of Belief Revision.*

The semantics is formally expressed with the following set of definitions, revised from [2] to make it simpler, precise, and to comply even more with the postulates, which is part of the *contribution of this paper* and the difference with the one in [2] and the extended abstract in [4]. So, let us start with some definitions.

An *α-relaxed rule* is a rule ρ that is *weakened* by a default-negated atom α in its body: $\mathsf{Head}(\rho) \leftarrow \mathsf{Body}(\rho) \cup \{\neg\alpha\}$. In addition, an *$\alpha$-relaxed program* is a set of α-relaxed rules. On the other hand, a *generalised program* of $\mathcal{A}^*$ is a set of rules of form $\{\ell \leftarrow \top \mid \ell \in \mathcal{A}^*\}$, where $\mathcal{A}^*$ is a given set of literals.

Accordingly, updating a program with another consists in transforming an ordered pair of programs into a single abductive program, as follows.

Definition 7 ($\bullet$-update Program). *Given an updating pair of extended logic programs, denoted as $\Pi_1 \bullet \Pi_2$, over a set of atoms $\mathcal{A}$; and a set of unique abducibles $\mathcal{A}^*$, such that $\mathcal{A} \cap \mathcal{A}^* = \emptyset$; and the α-relaxed program Π' from Π_1, such that $\alpha \in \mathcal{A}^*$; and the abductive program $\Pi_{\mathcal{A}^*} = \langle \Pi' \cup \Pi_2, \mathcal{A}^* \rangle$. Its $\bullet$-update program is $\Pi' \cup \Pi_2 \cup \Pi_G$, where Π_G is a generalised program of $\mathcal{M} \cap \mathcal{A}^*$ for some minimal generalised answer set $\mathcal{M}$ of $\Pi_{\mathcal{A}^*}$ and "$\bullet$" is the corresponding update operator.*

Obviously Definition 7 allows none or more $\bullet$-*update programs.* In addition to that, Corollary 2 below shows that the update is always consistent provided that Π_1 is also consistent. Before that, let us formalise another minor obvious property:

Corollary 1. *Let Π_G be a generalised program out of a minimal generalised answer set $\mathcal{M}$ from $\Pi_{\mathcal{A}^*}$ and $\mathcal{M}_1$ an answer set of Π_G. The following two statements hold:*

a) $\mathcal{M}_1 = \mathcal{M} \cap \mathcal{A}^*$.
b) $\mathcal{M}_1 \subseteq \mathcal{M}$.

Last but not least, the associated *models $\mathcal{S}$ of the new knowledge base* correspond to the answer sets of a •-update program as follows.

Definition 8 (•-update Answer Set). *Let* $\mathbf{\Pi_\bullet} = (\Pi_1 \bullet \Pi_2)$ *be an update pair over a set of atoms* $\mathcal{A}$. *Then,* $\mathcal{S} \subseteq \mathcal{A}$ *is a* •-answer set of $\mathbf{\Pi_\bullet}$ *if and only if* $\mathcal{S} = \mathcal{S}' \cap \mathcal{A}$ *for some* minimal generalised answer set $\mathcal{S}'$ *of its* •-update program.

Intuitively, this formulation establishes an order with respect to the *latest update* —which corresponds to Katsuno and Mendelzon [17, 18]'s first postulate (R∘1)— and with respect to a *minimal change* when choosing the most preferred model: MGAS.

5 •-Properties

The following sets of properties of this simpler formulation are the main contribution of this current semantics for *iterated updates* of *epistemic states*. They are classified into a study of consistency issues, and the satisfaction itself of KM′-postulates.

Before the main results, there are two particular properties suggested much earlier in [23] that are necessary for the rest of them. Additionally, the reader should note that a statement like $\Pi_1 \equiv \Pi_2$ means that both Π_1 and Π_2 have the same answer sets —or alternately $\Pi_1 \equiv_{\mathsf{ASP}} \Pi_2$. By a slight abuse of notation, when establishing equivalence between updates, indeed it means that they have the same (or different) update answer sets. Finally, the two properties from the literature (ref. [19, 20]), interpreted in our own notation, are the following.

•-SP-8, Strong Consistency, SC: If $\Pi_1 \cup \Pi_2$ is *consistent*, then $\Pi_1 \bullet \Pi_2 \equiv \Pi_1 \cup \Pi_2$.
    The update coincides with the union when $\Pi_1 \cup \Pi_2$ is consistent.
•-SP-9, Weak Irrelevance of Syntax, WIS: Let Π, Π_1, and Π_2 be logic programs under the same language. If $T_{\mathcal{N}_2}(\Pi_1) \equiv_{\mathcal{N}_2} T_{\mathcal{N}_2}(\Pi_2)$ then $\Pi \bullet \Pi_1 \equiv \Pi \bullet \Pi_2$.

Theorem 1. *[[2]] Suppose that Π, Π_1, Π_2 and Π_3 are* ELP. *Operator* • *satisfies the properties* •-SP-8 *and* •-SP-9.

This preliminary result shall be helpful to simplify further properties in upcoming sections.

5.1 Dealing with Inconsistencies

As previously suggested, dealing with inconsistencies is something necessary not only when new information contradicts previous one, but also with an *originally-inconsistent knowledge base*. This section consists of a study of consistency-preservation and consistency-restoration as key properties of •-operator.

In particular, *Weak-consistency View* guarantees consistency of the abductive program from an update pair. On the other hand, a *consistent abductive program* from a •-update pair shall mean the abductive program with generalised answer sets.

Lemma 1 (Weak Consistency View). *Suppose Π_0 and Π_1 are ELP's and an updating pair $\Pi_0 \bullet \Pi_1$ with its corresponding abductive program $\Pi_{\mathcal{A}^*} = \langle\, \Pi' \cup \Pi_1, \mathcal{A}^*\,\rangle$. If Π_1 is consistent then $\Pi_{\mathcal{A}^*}$ is also consistent.*

Accordingly, the following result holds.

Corollary 2 (Consistency Preservation). *Suppose Π_0 and Π_1 are ELP's. The update $\Pi_0 \bullet \Pi_1$ is consistent if Π_1 is consistent.*

Proof. The proof is similar to the one in Lemma 1

This property is known in the literature as *Consistency Preservation* and by Sakama and Inoue as *Inconsistency Removal*. Note that the sole name of the latter confirms the *syntactical orientation* of their approach. Last, it's wort noticing that this property is equivalent to postulate (R ∘ 3) both Katsuno and Mendelzon's and Darwiche and Pearl's third postulate.

On the other hand, Π_1 inconsistent in Corollary 2 may lead to two possible situations: that the resulting update is either consistent or inconsistent, as shown in the following example.

Example 1 (Inconsistent Update). Suppose the update $\Pi_1 = \{a \leftarrow \neg a\}$, which has no answer sets, to an original *empty knowledge base* $\Pi_0 = \emptyset$. As a result, $\bot \models \Pi_0 \bullet \Pi_1$. Now suppose the same update to an initial knowledge base $\Pi'_0 = \{a \leftarrow \top\}$. The •-update answer set of such an update $\{a\} \models \Pi'_0 \bullet \Pi_1$.

Corollary 2 also proves to be useful both when satisfying belief revision postulates and when *restoring consistency* from an originally inconsistent knowledge base, as explained below. On top of that, this property is a general case of Sakama and Inoue's *inconsistency removal* that makes syntactical changes to the original knowledge base.

Next, the following proposition follows directly from Corollary 2.

Proposition 1 (Consistency Restoration). *Suppose Π_0 is an ELP. The update $\Pi_0 \bullet \emptyset$ is consistent.*

Proof. The proof is similar to the one in Lemma 1

As described in this section, •-operator guarantees *robustness of knowledge bases* in many situations where other alternate frameworks break down. Accordingly, the properties presented in this section shall be part of a more general framework of principles and postulates.

Next, the core of this paper is the introduction of a particular interpretation of one of the latest formulations of the AGM-postulates (KM′) and which of them are met by •-operator.

5.2   Principles

One of the main goals of this paper is a formulation of a semantics for updates of logic programs that can meet as-many-as possible general properties. So, let us start this section with an interpretation and characterisation of Katsuno and Mendelzon's postulates $(\mathsf{R} \circ 1)$–$(\mathsf{R} \circ 6)$, as a main foundation to this *revision* of *epistemic states*.

$(\mathsf{RG} \circ 1)$ $\Pi_1 \subseteq \Pi \circ \Pi_1$.
$(\mathsf{RG} \circ 2)$ If $\Pi \cup \Pi_1$ is consistent, then $\Pi \circ \Pi_1 \equiv_{\mathsf{ASP}} \Pi \cup \Pi_1$.
$(\mathsf{RG} \circ 3)$ If Π_1 is consistent, then $\Pi \circ \Pi_1$ is also consistent.
$(\mathsf{RG} \circ 4)$ If $\Pi_x = \Pi_y$ and $\Pi_1 \equiv_{\mathcal{N}_2} \Pi_2$ then $\Pi_x \circ \Pi_1 \equiv_{\mathsf{ASP}} \Pi_y \circ \Pi_2$.
$(\mathsf{RG} \circ 4')$ If $\Pi_1 \equiv_{\mathcal{N}_2} \Pi_2$ then $\Pi \circ \Pi_1 \equiv_{\mathsf{ASP}} \Pi \circ \Pi_2$.
$(\mathsf{RG} \circ 5)$ $\Pi \circ (\Pi_1 \cup \Pi_2) \subseteq (\Pi \circ \Pi_1) \cup \Pi_2$.
$(\mathsf{RG} \circ 6)$ If $(\Pi \circ \Pi_1) \cup \Pi_2$ is consistent, then $(\Pi \circ \Pi_1) \cup \Pi_2 \subseteq \Pi \circ (\Pi_1 \cup \Pi_2)$.

An immediate result is the following main theorem of this paper certifying that $\bullet$-operator satisfies five of these six belief revision postulates.

Theorem 2 ($\mathsf{RG} \circ$ -properties). *Suppose that Π, Π_1 and Π_2 are* ELP. *Update operator "$\bullet$" satisfies properties* $(RG \circ 1)$–$(RG \circ 4)$ *and* $(RG \circ 6)$.

Proof. $(\mathsf{RG} \circ 1)$ $\Pi_1 \subseteq \Pi \bullet \Pi_1$.
By Definition 7, $\Pi_1 \subseteq \Pi' \cup \Pi_G \cup \Pi_1$ that clearly satisfies the objective.
$(\mathsf{RG} \circ 2)$ If $\Pi \cup \Pi_1$ is consistent, then $\Pi \bullet \Pi_1 \equiv \Pi \cup \Pi_1$.
This postulate corresponds to Strong-Consistency property and satisfied by Theorem 1.
$(\mathsf{RG} \circ 3)$ If Π_1 is consistent, then $\Pi \bullet \Pi_1$ is also consistent.
This postulate is equivalent to Corollary 2.
$(\mathsf{RG} \circ 4')$ If $\Pi_1 \equiv_{\mathcal{N}_2} \Pi_2$ then $\Pi \bullet \Pi_1 \equiv \Pi \bullet \Pi_2$.
This postulate is equivalent to property $\bullet$-SP-9 and satisfied by Theorem 1.
$(\mathsf{RG} \circ 6)$ If $(\Pi \bullet \Pi_1) \cup \Pi_2$ is consistent, then $(\Pi \bullet \Pi_1) \cup \Pi_2 \subseteq \Pi \bullet (\Pi_1 \cup \Pi_2)$.
Suppose $(\Pi \bullet \Pi_1) \cup \Pi_2$ is consistent. Then, the abductive program of $\Pi \bullet \Pi_1$ is $\langle \Pi' \cup \Pi_1, \mathcal{A}^* \rangle$ with its respective MGAS's $\mathcal{M}(\Delta_1)$ that is an answer set of $\Pi' \cup \Pi_1 \cup \{\alpha \leftarrow \top | \alpha \in \Delta_1\}$ where $\Delta_1 \subseteq \mathcal{A}^*$ and its corresponding generalised program Π_{G_1}. By Definition 7, the update $\Pi \bullet (\Pi_1 \cup \Pi_2)$ has the abductive program $\langle \Pi' \cup (\Pi_1 \cup \Pi_2), \mathcal{A}^* \rangle$ with its MGAS's $\mathcal{M}(\Delta_2)$ that is an answer set of $\Pi' \cup \Pi_1 \cup \Pi_2 \cup \{\alpha \leftarrow \top \mid \alpha \in \Delta_2\}$ where $\Delta_2 \subseteq \mathcal{A}^*$ and its corresponding generalised program Π_{G_2}. Because Π_2 is consistent with the update $\Pi \bullet \Pi_1$, the number of abducibles in Δ_1 never change, and it's easy to verify that Δ_2 contains at least the same abducibles than Δ_1, $\Delta_1 \subseteq \Delta_2 \subseteq \mathcal{A}^*$ and thus $\Pi_{G_1} \subseteq \Pi_{G_2}$. In consequence, the respective $\bullet$-update programs of each pair are $\Pi' \cup \Pi_1 \cup \Pi_2 \cup \Pi_{G_1}$ and $\Pi' \cup \Pi_1 \cup \Pi_2 \cup \Pi_{G_2}$, where $\Pi' \cup \Pi_1 \cup \Pi_2 \cup \Pi_{G_1} \subseteq \Pi' \cup \Pi_1 \cup \Pi_2 \cup \Pi_{G_2}$ as required. Therefore, $(\Pi \bullet \Pi_1) \cup \Pi_2 \subseteq \Pi \bullet (\Pi_1 \cup \Pi_2)$.

Nevertheless, postulate $(\mathsf{RG} \circ 5)$ does not hold. As a counterexample, consider the following programs: $\Pi = \{a \leftarrow \top; \sim b \leftarrow \top; \sim c \leftarrow \top\}; \Pi_1 = \{b \leftarrow \top\}; \Pi_2 = \{c \leftarrow \top\}$. Such an update inverts the direction of the relation.

5.3   Discussion

This section is an introduction to new general properties characterising •-operator that go from the structural properties, most of them inherited from its equivalent counterpart in [23], to more general ones encoded in our particular interpretation of belief revision postulates. The satisfaction of AGM-postulates in ASP is something new and important, provided that other current approaches either don't meet most of them or have discarded them for considering that their *monotonic nature* is incompatible with non-monotonic frameworks like ASP, as previously discussed in Section 2.

Another issue other approaches have is when updating in a sequence rather than an iterated fashion, which leads to counterintuitive results, especially in the persistence situation and when new updates arise afterwards. By following the original AGM paradigm, we also claim that the iterative approach has other more natural properties than its sequenced counterpart.

The section is also a *study of inconsistencies* not only due to new information that contradicts current knowledge, but also from both an *originally-inconsistent knowledge base*, as well as *new originally-inconsistent observations* that not necessarily contradict current beliefs. The former is something that may be considered a key feature of belief revision. However, as one of the main goals of this paper is to provide a strong general framework to correctly represent knowledge, and making a strict distinction with belief update theory might result controversial.

On the other hand, dealing with *originally-inconsistent observations* might seem counterintuitive to some researches, but it does not mean that observing such contradictions may not be possible in a *changing environment*. Take for example two *concurrent observations* that contradict each other, updating a current knowledge base in, say, a problem of Ambient Intelligence when a sensor fails and another one contradicts it. Another example is an *observation that is inconsistent* due to a typo or another kind of human error. Traditionally, those problems are left to future debugging, but with a tendency to model even-more *autonomous entities*, tolerating inconsistencies is not only reasonable, but also necessary to preserve a knowledge base from collapse.

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